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# EFFECT OF BROKEN ICE ON THE WAVE RESISTANCE OF AN AMPHIBIAN AIR-CUSHION VEHICLE IN NONSTATIONARY MOTION

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*The uniformly accelerated motion of an amphibian air-cushion vehicle on the surface of a basin covered by finely small ice floes is considered.*

**1.** The hydrodynamic problem of an amphibian air-cushion vehicle (AACV) that moves under ice conditions is modeled by a system of surface pressures [1] moving over the weighable free surface of a floating fluid [2–4].

We consider an infinite field covered by broken ice over which a given system of surface pressures $q$ moves with velocity $u(t)$. The coordinate system aligned with the vehicle is positioned in the following way: the $xOy$ plane coincides with the unperturbed ice–water interface, the $x$ axis is along the motion of the vehicle, and the $z$ axis is directed vertically upward. The water is an ideal incompressible fluid of density $\rho_2$, and the fluid motion is potential. The surface density of the floating fluid is set by a continuous function $m(x,y)$ [3]:

$$m(x,y) = \rho_1(x,y)h(x,y) = \rho_1^0 s_1(x,y)h(x,y),$$

where $\rho_1(x,y)$ is the ice density smeared out over the fluid surface, $\rho_1^0$ is the physical density of ice, $s_1(x,y)$ is a dimensionless function of ice-floe tightness ($0 \leqslant s_1 \leqslant 1$), and $h(x,y)$ is the ice thickness. It is assumed that $\rho_1$ and $h$ are constant quantities.

According to [1, 5], the wave resistance acting on the AACV is calculated by the formula

$$R = \iint\limits_{(\Omega)} q\,\frac{\partial w}{\partial x}\,dx\,dy, \tag{1.1}$$

where $\Omega$ is the region of load distribution $q(x,y,t)$ and $w(x,y,t)$ is the floating-fluid deformation surface; in the linear theory of waves, in a specific coordinate system this surface is determined as

$$w = -\frac{q}{\rho_2 g} - \frac{1}{g}\frac{\partial\Phi}{\partial t}\Big|_{z=0} + \frac{u}{g}\frac{\partial\Phi}{\partial x}\Big|_{z=0} - \frac{\rho_1 h}{\rho_2 g}\frac{\partial^2\Phi}{\partial t\,\partial z}\Big|_{z=0} + \frac{u\rho_1 h}{\rho_2 g}\frac{\partial^2\Phi}{\partial x\,\partial z}\Big|_{z=0}. \tag{1.2}$$

Here $u$ is the velocity of the system of surface pressures.

The desired function of the velocity potential $\Phi(x,y,z,t)$ should satisfy the Laplace equation $\Delta\Phi = 0$ and the linearized boundary conditions

$$m\Big(\frac{\partial^3\Phi}{\partial t^2\,\partial z} - u'_t\,\frac{\partial^2\Phi}{\partial x\,\partial z} - 2u\,\frac{\partial^3\Phi}{\partial t\,\partial x\,\partial z} + u^2\frac{\partial^3\Phi}{\partial z\,\partial x^2}\Big)$$

$$= -\rho_2 g\,\frac{\partial\Phi}{\partial z} - \rho_2\Big(\frac{\partial^2\Phi}{\partial t^2} - u'_t\,\frac{\partial\Phi}{\partial x} - 2u\,\frac{\partial^2\Phi}{\partial t\,\partial x} + u^2\frac{\partial^2\Phi}{\partial x^2}\Big) - \frac{\partial q}{\partial t} + u\,\frac{\partial q}{\partial x} \quad \text{for} \quad z = 0, \tag{1.3}$$

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$$\frac{\partial\Phi}{\partial z} = 0 \quad \text{for} \quad z = -H.$$

Here $H = H_1 - b$, where $H_1$ is the depth of the basin and $b = \rho_1 h/\rho_2$ is the depth of ice immersion in static equilibrium. For large depths, when $H_1$ is greater than $h$, one can assume that $H \approx H_1$.

Provided that the vehicle is idle at the moment $t = 0$ and there are no perturbations, except for the static deformation of the free surface, the initial conditions for the function $\Phi(x, y, z, t)$ are written in the form

$$\frac{\partial\Phi}{\partial z}\Big|_{z=0,\,t=0} = 0, \qquad \Big(\frac{\partial\Phi}{\partial t} + \frac{\rho_1 h}{\rho_2}\frac{\partial^2\Phi}{\partial z\,\partial t}\Big)\Big|_{z=0,\,t=0} = 0. \tag{1.4}$$

**2.** Let the pressure $q$ not depend on time in the specified moving coordinate system, i.e., $q = q(x, y)$. In addition, it is assumed that the functions $\Phi(x, y, z, t)$ and $q(x, y)$ satisfy the conditions necessary to represent them as an expansion into Fourier integrals in two variables $x$ and $y$. According to [2], we write

$$\Phi(x, y, z, t) = \frac{1}{4\pi^2}\int_0^\infty k\,dk \int_{-\pi}^{\pi} d\theta \iint_{(\Omega)} (F\exp(-kz) + E\exp(kz))$$

$$\times \exp(ik((x - x_1)\cos\theta + (y - y_1)\sin\theta))\,dx_1\,dy_1, \tag{2.1}$$

$$q(x, y) = \frac{1}{4\pi^2}\int_0^\infty k\,dk \int_{-\pi}^{\pi} d\theta \iint_{(\Omega)} q(x_1, y_1)\exp(ik((x - x_1)\cos\theta + (y - y_1)\sin\theta))\,dx_1\,dy_1,$$

where $F$ and $E$ are desired functions of the variables $x_1$, $y_1$, $t$, $k$, and $\theta$.

Substitution of the expressions (2.1) into the boundary conditions (1.3) allows one to obtain the dependence between $F$ and $E$ and the differential equation for $F$:

$$E = F\exp(2kH),$$

$$F''_{tt} - 2F'_t u\mu + Fu^2\mu^2 - Fu'_t\mu = -\frac{\rho_2 gFk\tanh(kH)}{\rho_1 hk\tanh(kH) + \rho_2} + \frac{uq\mu}{(1 + \exp(2kH))(\rho_1 hk\tanh(kH) + \rho_2)}, \tag{2.2}$$

$$\mu = ik\cos\theta.$$

To solve Eq. (2.2), by analogy with [6], the function

$$F_1 = F\exp(-\mu s) \tag{2.3}$$

is introduced into the consideration. Here $s(t) = \int_0^t u(\tau)\,d\tau$ is the distance passed by the AACV for the time $t$.

Substituting (2.3) into (2.2) in the solution of Eq. (2.2) with the use of the initial conditions (1.4) results in the following expression for $F_1$:

$$F_1 = \frac{\sin(\beta_1 t)}{\beta_1}\int_0^t f(\tau)\cos(\beta_1\tau)\,d\tau - \frac{\cos(\beta_1 t)}{\beta_1}\int_0^t f(\tau)\sin(\beta_1\tau)\,d\tau,$$

$$\beta_1 = \sqrt{\frac{\rho_2 gk\tanh(kH)}{\rho_1 hk\tanh(kH) + \rho_2}}, \qquad f(\tau) \equiv \frac{u(\tau)q(x_1, y_1)\mu\exp(-\mu s(\tau))}{(1 + \exp(2kH))(\rho_1 hk\tanh(kH) + \rho_2)}. \tag{2.4}$$

Substituting the resulting dependences (2.2)–(2.4) for $E$ and $F$ into the expression for the velocity potential (2.1), using formulas (1.1) and (1.2), and replacing the variables $k = \lambda$ and $k\cos\theta = \alpha$, after simple transformations one obtains a formula in the general form for the wave resistance of the system of surface pressures $q(x,y)$ upon nonstationary motion over the free surface of a floating fluid:

$$R = -\frac{1}{\rho_2 g} \iint\limits_{(\Omega)} q \frac{\partial q}{\partial x}\, dx\, dy + \frac{1}{2\rho_2 g \pi^2} \int\limits_0^t u(\tau) \cos(\beta_1(t-\tau))\, d\tau$$

$$\times \int\limits_0^\infty \lambda\, d\lambda \int\limits_{-\lambda}^{\lambda} \cos(\alpha(s(t)-s(\tau)))(C_1^2 + C_2^2 + C_3^2 + C_4^2) \frac{\alpha^2\, d\alpha}{\sqrt{\lambda^2-\alpha^2}},$$

$$C_1 = \iint\limits_{(\Omega)} q(x,y) \cos(x\alpha) \cos(y\sqrt{\lambda^2-\alpha^2})\, dx\, dy, \tag{2.5}$$

$$C_2 = \iint\limits_{(\Omega)} q(x,y) \cos(x\alpha) \sin(y\sqrt{\lambda^2-\alpha^2})\, dx\, dy,$$

$$C_3 = \iint\limits_{(\Omega)} q(x,y) \sin(x\alpha) \cos(y\sqrt{\lambda^2-\alpha^2})\, dx\, dy,$$

$$C_4 = \iint\limits_{(\Omega)} q(x,y) \sin(x\alpha) \sin(y\sqrt{\lambda^2-\alpha^2})\, dx\, dy, \qquad \beta_1 = \sqrt{\frac{\rho_2 g \lambda \tanh(\lambda H)}{\rho_1 h \lambda \tanh(\lambda H) + \rho_2}}.$$

If a rectangular or elliptic, in plan, system of constant pressures $q(x,y) = q_0 \equiv \text{const}$ is taken as a system of moving pressures $q(x,y)$, the theoretically obtained curve of wave resistance [1, 2, 5, 6] has an infinite number of vibrations in the region of small velocities. This result is not supported by experimental data. This drawback of the theory can be overcome by introducing a system of pressures $q(x,y)$ that is described by means of the function of hyperbolic tangent [6]:

$$q(x,y) = \frac{q_0}{4}\Big[\tanh\Big(\alpha_1\Big(x+\frac{L}{2}\Big)\Big) - \tanh\Big(\alpha_1\Big(x-\frac{L}{2}\Big)\Big)\Big]$$

$$\times\Big[\tanh\Big(\alpha_2\Big(y+\frac{L}{2\omega}\Big)\Big) - \tanh\Big(\alpha_2\Big(y-\frac{L}{2\omega}\Big)\Big)\Big]. \tag{2.6}$$

Here $q_0$ is the nominal pressure, $L$ is the vehicle length, $\omega = L/B$ is the elongation of the vehicle, $B$ is the vehicle width, and $\alpha_1$ and $\alpha_2$ the parameters that describe the degree of deviation of the pressure distribution in the longitudinal and transversal directions from the rectangular form. The greater $\alpha_1$ and $\alpha_2$ in magnitude, the closer the form of the pressure distribution to a rectangle. As $\alpha_1$, $\alpha_2 \to \infty$, we have $q$ equivalent to $q_0$ uniformly distributed over the rectangle. For better agreement between the theoretical and experimental results, Doctors and Sharma [6] proposed to use $\alpha_1 L = \alpha_2 L = 10$.

For the uniformly accelerated motion of the specified system of pressures (2.6), the expression (2.5) takes the form

$$R/D = A q_0/(\rho_2 g L),$$

$$A(k_L, k_a, \varepsilon, \omega, \gamma) = \frac{\pi^2\omega}{(\alpha_1 L)^2(\alpha_2 L)^2 k_L k_a} \int\limits_0^1 \tau \cos\Big(\frac{1-\tau}{k_L k_a} \sqrt{\frac{k_L \lambda\, \mathrm{th}(\gamma\lambda)}{\varepsilon\lambda \tanh(\gamma\lambda)+1}}\Big) d\tau \tag{2.7}$$

$$\times \int_0^\infty \lambda\, d\lambda \int_0^\lambda \cos\left(\frac{\alpha(1-\tau^2)}{k_L k_a}\right) \frac{\sin^2(\alpha/2)\sin^2(\sqrt{\lambda^2-\alpha^2}/(2\omega))}{\sinh^2(\pi\alpha/(2\alpha_1 L))\sinh^2(\pi\sqrt{\lambda^2-\alpha^2}/(2\alpha^2 L))\sqrt{\lambda^2-\alpha^2}}\, \alpha^2\, d\alpha,$$

$$D = q_0 LB, \quad k_L = \frac{gL}{u^2}, \quad \varepsilon = \frac{\rho_1 h}{\rho_2 L}, \quad \omega = \frac{L}{B}, \quad \gamma = \frac{H}{L}, \quad k_a = \frac{a}{g}, \quad u(t) = at.$$

In contrast to (2.5), here the variables of integration are dimensionless.

Using the results of [2] and the system of pressures (2.6), for the wave-resistance coefficient of the AACV during its stationary motion over the floatation-fluid surface, one can derive the formula

$$A(k_L, \varepsilon, \omega, \gamma) = \frac{\pi^3\omega}{2(\alpha_1 L)^2(\alpha_2 L)^2} \int_{\lambda_0}^\infty \frac{\beta^2 \sin^2(\beta/2)\sin^2(\sqrt{\lambda^2-\beta^2}/(2\omega))\,\lambda\, d\lambda}{\sqrt{\lambda^2-\beta^2}\,\sinh^2(\pi\beta/(2\alpha_1 L))\sinh^2(\pi\sqrt{\lambda^2-\beta^2}/(2\alpha_2 L))},$$

$$\beta = \sqrt{\frac{k_L\lambda\tanh(\gamma\lambda)}{1+\varepsilon\lambda\tanh(\gamma\lambda)}}. \tag{2.8}$$

Here $\lambda_0$ is the solution of the transcendental equation $k_L \tanh(\gamma\lambda) = \lambda(1+\varepsilon\lambda\tanh(\gamma\lambda))$, $u = \text{const}$, and the other notion corresponds to formula (2.7).

**3.** The results of the numerical calculations by formulas (2.7), (2.8) were compared with known theoretical results. For a zero flotation parameter ($\varepsilon = 0$), the wave-resistance curve calculated by formulas (2.7) and (2.8) agrees with the results of [6], which are plotted as diagrams, for all the depth, elongation, and acceleration parameters, which were considered in [6], namely, for $\gamma = 0.25$ and $\gamma = \infty$ and $\omega = 2$; $k_a = 0.05$ and $k_a = 0.1$, and the stationary motion. For the flotation parameters $\varepsilon = 0.045$ and $\varepsilon = 0$, the results obtained by means of formula (2.8) were compared with the numerical results of [2], in which the stationary motion of a rectangular system of constant pressures was studied. The effect of the form of the cumulative distribution function of surface pressure $q(x,y)$ on the magnitude of the maximum wave resistance was analyzed. It was found that if the function $q(x,y)$ is defined by formula (2.6) with the coefficients $\alpha_1 L = \alpha_2 L = 10$ for any $\varepsilon$, $\omega$, and $\gamma$, the maximum value of $A$ exceeds the corresponding value from [2] by 18–20% in average. Here the form of the pressure function does not affect the value of the critical number $k_L^*$ at which the maximum wave resistance to the AACV is reached. As $\alpha_1 L$ and $\alpha_2 L$ are increased, the results of calculations for the maximum value of the coefficient $A$ by formula (2.8) tend to the corresponding values in [2] and coincide with an accuracy of 2% already for $\alpha_1 L = \alpha_2 L = 50$.

The main results of the calculations by formulas (2.7) and (2.8) are given in Figs. 1–4. Figures 1 and 2 show the maximum value of the wave-resistance coefficient $A$ (the quantity $A^*$) versus the elongation of the vehicle $\omega$ for $\varepsilon = 0$ and 0.045, respectively. Here curves 1–9 refer to the stationary case ($k_a = 0.05$ and 0.10): curves 1–3 for $\gamma = \infty$, curves 4–6 for $\gamma = 0.5$, and curves 7–9 for $\gamma = 0.25$. One can see from Figs. 1 and 2 that the insignificant decrease in the maximum value of the wave-resistance coefficient occurs for deep water and great elongation with acceleration. The effect of acceleration on $A^*$ is enhanced with decrease in depth $\gamma$ and elongation $\omega$. The flotation decreases the maximum wave resistance.

Figures 3 and 4 show values of $k_L^*$ at which the vehicle undergoes the maximum wave resistance for $\varepsilon = 0$ and 0.045, respectively. Here curves 1–3 refer to $\gamma = \infty$, 0.5, and 0.25 for $k_a = 0.1$, curves 4–6 to $\gamma = \infty$, 0.5, and 0.25 for $k_a = 0.05$, and curves 7–9 to $\gamma = \infty$, 0.5, and 0.25 for the stationary case. It is noteworthy that, for the stationary problem, the maximum wave resistance occurs when the vehicle moves with a constant critical velocity $u = u^*$; for uniformly accelerated motion, the maximum of $R$ corresponds to the moment $t$, when the instantaneous velocity of the vehicle is $u = at = u^*$. As can be seen from Figs. 3 and 4, the flotation leads to the shift of $k_L^*$ toward smaller velocities by 13% on average for any $\omega$ and $\gamma$ in uniformly accelerated motion and in the stationary case. The growth of acceleration results in a significant displacement of the point of maximum wave resistance toward large velocities; the more shallow the water, the greater the effect of acceleration.

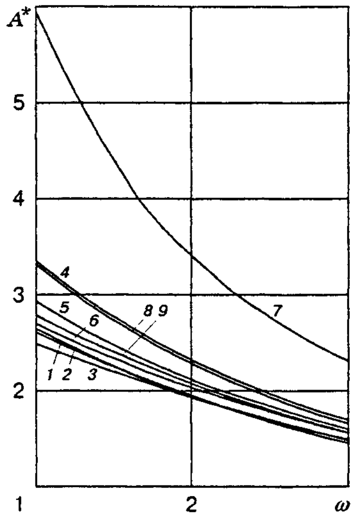


Fig. 1

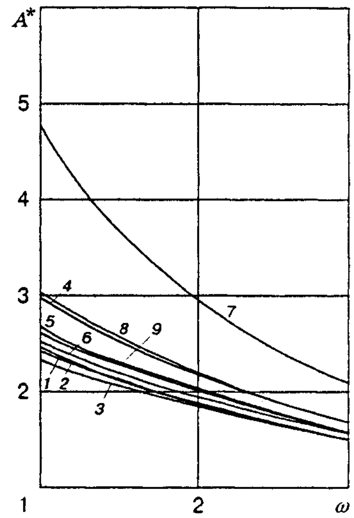


Fig. 2

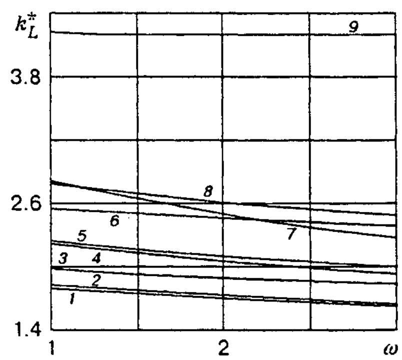


Fig. 3

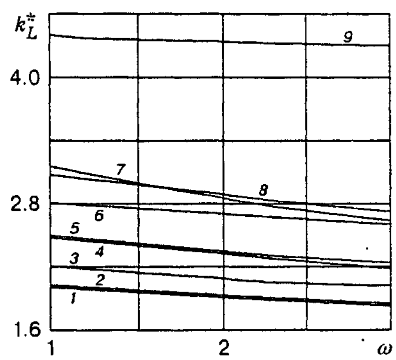


Fig. 4

Analyzing the numerical results, to calculate the critical $k_L^*$ in the range $1 \leqslant \omega \leqslant 3$, $\gamma \geqslant 0.5$, and for $\varepsilon = 0$, one can use the formulas for $k_L^* = 3.01 - 14.7k_a + 34k_a^2 - (0.21 - 1.3k_a)\omega$ for $0 < k_a \leqslant 0.1$ and $k_L^* = 3.01 - 0.21\omega$ in the stationary case; the error of the calculations performed with the use of these values is smaller than 5% compared to the numerical results.

Along with calculations of the wave resistance of the AACV upon motion over the field of small ice floes, our studies allow one to give recommendations on the choice of the modes of motion of AACVs employed under the conditions of navigable canals in continuous ice. The calculations show that to prevent breaking off of the canal edges in the sites with a thin ice cover and encumbering of the latter by large ice floes hindering the motion of displacing vehicles, AACVs must move with maximum acceleration.

The resulting dependences are of interest for the solution of the inverse problem as well. As follows from theoretical calculations, to expand the broken-ice field or to refine ice floes in the previously built canal, the AACV must move in a stationary regime with velocities recommended depending on the vehicle parameters and ice situation.

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